

# Logarithms

## 1. Basic Logarithmic Formula

A logarithm tells you the power to which a number (base) must be raised to get another number. The general formula is:

$$\log_b(x)=y$$

This equation reads as "log base b of x is y," which means that b raised to the power of y equals x.

For example:

$$\log_{10}(100)=2$$

$$\text{since } 10^2=100$$

## 2. Types of Logarithms

- **Common Logarithm:** Base 10, written as  $\log(x)$  or  $\log_{10}(x)$ .
- **Natural Logarithm:** Base e (Euler's number, approximately 2.718), written as  $\ln(x)$  or  $\log_e(x)$ .

## 3. Calculating Logarithms by Hand

For simple logarithms with base 10 or base e, you can use a calculator. However, here are some ways to estimate or calculate them:

### For Common Base 10 Logarithms

1. **Perfect Powers of 10:** Use known values. For example:
  - $\log_{10}(10)=1$  because  $10^1=10$ .
  - $\log_{10}(1000)=3$  because  $10^3=1000$ .
2. **Estimating Non-Perfect Powers:** For values like  $\log_{10}(50)$ , you can use a calculator, or estimate by knowing it's between  $\log_{10}(10)=1$  and  $\log_{10}(100)=2$ .

### For Natural Logarithms

Use a calculator for precise values, but remember:

- $\ln(e)=1$  because  $e^1=e$ .
- $\ln(1)=0$  because  $e^0=1$ .

## 4. Using Logarithmic Rules

To simplify calculations, apply these rules:

- **Product Rule:**  $\log_b(x \cdot y)=\log_b(x)+\log_b(y)$
- **Quotient Rule:**  $\log_b(x/y)=\log_b(x)-\log_b(y)$
- **Power Rule:**  $\log_b(x^n)=n \cdot \log_b(x)$
- **Change of Base Formula:** For any bases a and b,

$$\log_b(x)=\log_a(x)/\log_a(b)$$